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A MIXED AEROSOL—PARTICLE CHARGE. THE ASYMPTOTE AND INTERPOLATION FORMULAS FOR THE ELECTRIFICATION CURRENT

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In the electrohydrodynamic flows of weakly ionized aerosols particles may undergo electrification due to a combination of ion charges [1]. With a low disperse-phase concentration we can limit ourselves in the description of this process to a study of the charge of a single particle. The present study is devoted to an investigation of a mixed charge, where diffusion significantly affects ion motion in the electric field generated by external forces in the vicinity of the particle. In studying this mixed charge, as a rule, we can neglect the motion of the gas relative to the particle. In extreme cases in which neither the diffusion of the ions nor the external electric field have been taken into consideration, the problem of the unipolar charge of a spherical particle in a nonmoving weakly ionized gas has been solved in [2, 3]. The solution of the problem with respect to the influence exerted by a weak external electric field on the diffusion charge of the particle has been derived in [4]. In the present paper we examine the opposite case of a strong external electric field. We have used the method of joined asymptotic expansions [5] to find the distribution of ions in the vicinity of the particle as well as an expression for the electrification current, which refines the familiar solution [2]. These results are subsequently used in the construction of an approximate interpolation formula for the global electrification current. We note that the conventional summation of the limit expressions [2, 3] to calculate the electrification current in the case of a mixed charge produces major errors. Comparison with the results from a numerical solution of the problem on a computer shows that the constructed interpolation formula provides good approximation in the case of arbitrary values for the electric Peclet number Pe_E.

1. In disperse media consisting of a weakly ionized gas and dispersed particles, the latter may become charged by capturing the charge of the ions. Given a sufficiently small particle concentration, in order to study this phenomenon we will examine the electrification of a single ideally conducting spherical particle in a unipolar charged gas. Without loss of generality for the results, we will assume the ion charge to be positive. Let the ion concentration and the particle radius a be sufficiently small and we will assume the external electric field to be uniform at distances of $\sim a$.

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The effect of diffusion on the directed motion of the ion in the electric field is characterized by $Pe_E = a bE_0/D$, where $E_0 = |\mathbf{E}_0|$, where $\mathbf{E}_0$ is the strength of the external electric field, b and D are the coefficients of mobility and diffusion for the ions, associated by the Einstein relationship $b = eD/(kT)$, e is the proton charge, k is the Boltzmann constant, T is the absolute temperature. If $Pe_E \sim 1$, we have a mixed particle charge in whose calculation it is necessary simultaneously to make provision for ion diffusion and the external electric field. In numerous important cases it turns out that we have a small electric Reynolds number $Re_E = u/(bE_0)$ (u is the relative velocity of the particle) and we can neglect the motion of the gas. In the following the gas is assumed to be nonmoving relative to the particle.

Let K be the constant of the reaction rate for the transmission of the ion charge to the particle surface. Within the scope of these assumptions the volumetric charge density q will be determined in the vicinity of the particle through solution of the boundary-value problem:

$$\operatorname{div} \mathbf{j} = 0,\ \mathbf{j} = -D\nabla q + qb\mathbf{E},\ \mathbf{E} = -\nabla\varphi, \tag{1.1}$$
$$r = a\colon j_n = -Kq,\ r \to \infty\colon q \to q_0,$$
$$\varphi = -E_0 r\cos\theta\left(1 - \frac{a^3}{r^3}\right) + e_r\left(\frac{1}{r} - \frac{1}{a}\right).$$

Here r is the distance to the center of the particle; θ is the angle between the vector $\mathbf{E}_0$ and the radius vector of the point; $\mathbf{j}$ is the density of the electric current; q_0 is the unperturbed value of q; φ is the electric field potential. The subscript n identifies projections of the vectors onto the external normal to the surface S of the particle. We can find the global electrification current only after solving problem (1.1) by integration $I = -\int_S j_n\, ds$ and it depends both on q_0, E_0 and the particle charge e_r.

2. Boundary-value problem (1.1) has an analytical solution in the limit cases $Pe_E = 0$ [6] and $Pe_E = \infty$ [2]. In order to obtain an approximate formula for the function $I(E_0, e_r, q_0)$ for moderate Pe_E, let us take a look initially at the asymptote of the solution to problem (1.1) as $Pe_E \to \infty$.

We see that at the limit $Pe_E = \infty$ from (1.1) we have $E\nabla q = 0$, from which it follows that the volumetric charge density is constant along the force lines of the electric field strength. With $e_r{}^0 \equiv e_r/(3a^2E_0) \leq -1$ everywhere outside of the particle $q \equiv q_0$, and with $e_r{}^0 \sim -1$ the space outside of the particle is subdivided into two regions: a region with a zero value for the volumetric charge, filled with the lines of force emanating from the particle, and a region in which $q \equiv q_0$. The location of the boundary between these regions is governed by the magnitude of the dimensionless particle charge $e_r{}^0$. Along this boundary and also along that portion of the surface S, where $E_n < 0$, in the case of finite but large Pe_E a diffusion boundary layer is formed, so that the ions in the vicinity of the particle are distributed in rather complex fashion. In this connection, let us introduce the new dependent variable

$$w = (1 - q^*)\exp(\mathrm{Pe}_E\,\varphi^*/2),\ \varphi^* = \varphi/aE_0,\ q^* = q/q_0.$$

Problem (1.1) is transformed as follows:

$$\mathrm{Pe}_E^{-2}\left[\frac{1}{r^*}\frac{\partial^2 r^* w}{\partial r^{*2}} + \frac{1}{r^{*2}\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial w}{\partial\theta}\right)\right] - \frac{1}{4}E^{*2}w = 0, \tag{2.1}$$
$$r^* = 1\colon \frac{\partial w}{\partial r^*} = -\frac{1}{2}w\,\mathrm{Pe}_E E_n^* + (w-1)(E_n^* + K^*)\,\mathrm{Pe}_E,\ r^* \to \infty\colon w \to 0,$$
$$E^{*2} = \left[\frac{3e_r^0}{r^{*2}} + \cos\theta\left(1 + \frac{2}{r^{*3}}\right)\right]^2 + \sin^2\theta\left(1 - \frac{1}{r^{*3}}\right)^2,$$
$$r^* = r/a,\ E^* = E/E_0,\ K^* = K/bE_0.$$

At the limit $Pe_E = \infty$ it follows from (2.1) that w = 0 everywhere outside of the particle; with finite large values for the Peclet number and with any value for $e_r{}^0$ we have a significant change in w only in that layer adjacent to the particle surface, and this considerably simplifies our study.

The presence of the small parameter Pe_E^{-2} in the problem makes it possible to construct its approximate solution and, employing the relationship of the variables q^* and w, to find the distribution of the volumetric charge in the vicinity of the particle. The electrification currents are expressed directly in terms of the values of the function w and its derivative at the particle surface:

$$r^* = 1:\ j^0 = -\frac{1}{\mathrm{Pe}_E}\frac{\partial w}{\partial r^*} + E_n^*\left(\frac{w}{2}+1\right),$$

$$I^0 = \frac{1}{6}\int_0^\pi j^0 \sin\theta\, d\theta, \quad I^0 = \frac{I}{12\pi a^2 q_0 b E_0}, \quad j^0 = \frac{-j_n}{q_0 b E_0}.$$

Keeping in mind the symmetry of the problem, we will examine the behavior of the solution in the half-plane passing through the axis of symmetry and bounded by that axis. Comparative analysis of the individual terms in (2.1) when $\mathrm{Pe}_E \gg 1$, with provision made for the explicit form of the function E^2, demonstrates that it is possible to isolate several regions with diverse structures for the asymptotic solution. In each of these, from (2.1), we obtain an equation of simpler form for the principal terms of the expansion for the solution with respect to the small parameter.

In the outer region the solution of the problem $w = 0$ is found from differential equation (2.1) in which the terms with the factor Pe_E^{-2} are neglected.

In the boundary layer with the excluded region of the critical point of the vector field E: $0 \le r^* - 1 \le O(\mathrm{Pe}_E^{-1})$,

$$||e_r^0| - 1| \leqslant O(\mathrm{Pe}_E^{-2/3}):\ \sin\theta \geqslant O(\mathrm{Pe}_E^{-1/3}),$$
$$|e_r^0| \leqslant 1 - O(\mathrm{Pe}_E^{-2/3}):\ |\theta - \arccos(-e_r^0)| \geqslant O(\mathrm{Pe}_E^{-1/2}),$$
$$|e_r^0| \geqslant 1 + O(\mathrm{Pe}_E^{-2/3}):\ 0 \leqslant \theta \leqslant \pi$$

we will study the solution in the variables

$$y = \varepsilon^{-1}(r^* - 1),\ x = \cos\theta,\ \varepsilon = 2/3\ \mathrm{Pe}_E.$$

The coefficient in the function relating ε to the Peclet number has been introduced in order to achieve an equation of the simplest form.

Expanding the functions E^{*2} and r^{*-1} over the powers of ε and substituting into (2.1), for coefficients of the series $w = \sum_{n=0}^{\infty} w_n \varepsilon^n$ we have a chain of equations and boundary conditions:

$$\partial^2 w_0/\partial y^2 = \zeta^2 w_0,\ \zeta \equiv e^0_p + x = E_n^*/3, \tag{2.2}$$
$$\partial^2 w_1/\partial y^2 = \zeta^2 w_1 + 2A_0(|\zeta| - 2\zeta^2)\exp(-|\zeta| y)\ldots,$$
$$\zeta > 0:\ A_0 = 1,\ \zeta < 0:\ A_0 = 1 + 3\zeta/K^*,$$
$$y = 0:\ \frac{\partial w_0}{\partial y} = \zeta(w_0 - 2) + \frac{2}{3}K^*(w_0 - 1),\quad \frac{\partial w_m}{\partial y} = \left(\zeta + \frac{2}{3}K^*\right) w_m,$$
$$m \geqslant 1,\quad y \to \infty:\ w_p \to 0,\ p \geqslant 0.$$

As $y \to \infty$ the conditions must be satisfied for purposes of joining with the solution in the outer region. By solving (2.2) we will determine the principal terms of the expansion for the functions w and the local electrification current j^0 with respect to the small parameter, and turning to the variable q^*, we will obtain expressions for the distribution of the volumetric charge in the region under consideration:

$$w_0 = \exp(-|\zeta| y) A_0,\ w_1 = \zeta y^2 w_0,\ \zeta > 0: \tag{2.3}$$
$$q^* = O(\varepsilon^3),\ j^0 = O(\varepsilon^3),$$
$$\zeta < 0:\ q^* = (2\varepsilon y\zeta - 1)(1 + 3\zeta/K^*)\exp(2\zeta y) + 1 + O(\varepsilon^2),$$
$$j^0 = -3\zeta - \frac{9}{4K^*}\left[1 + \frac{1}{\zeta^2}\left(1 - e_r^{02}\right)\right]\varepsilon^2 + O(\varepsilon^3).$$

The density of the volumetric charge in the boundary layer when $E_n < 0$ and with moderate values of y is on the order of $q_0|E_n^*|/K^*$, and in the outer region on the order of $q \lesssim q_0$. In the formulation of problem (1.1) it was assumed that we could neglect the intrinsic electric field of the ions with an arbitrary value for $e_r{}^0$. For this, as follows from an estimate of the orders of the terms in the Maxwell equation div $E = 4\pi q$, we need the simultaneous satisfaction of the relationships $4\pi q_0 a/E_0 \ll 1$, $4\pi q_0 a/E_0 \ll K^* Pe_E$. Further, it is assumed that the condition $K^* \gtrsim 1$ is satisfied for the reaction rate constant.

Let us examine the vicinity of the singular point of the vector field for the case

$$|e^0_r| \leqslant 1 - O(\varepsilon^{2/3}),\ r^* - 1 \leqslant O(\varepsilon^{1/2}),\ |\theta - \arccos(-e^0_r)| \leqslant O(\varepsilon^{1/2}).$$

We will introduce the extended variables $X = (e_r{}^0 + \cos\theta)\varepsilon^{-1/2}$, $Y = (r^* - 1)\varepsilon^{-1/2}$. Substituting into (2.1) the expansions of the functions with respect to the small parameter $\varepsilon^{1/2}$, for the zeroth approximation of the function w we obtain the boundary-value problem in the upper half plane of the variables X, Y:

$$\frac{\partial^2 w_0}{\partial Y^2} + (1 - e^{02}_r)\frac{\partial^2 w_0}{\partial X^2} = [X^2 + (1 - e^{02}_r)Y^2]\,w_0, \tag{2.4}$$

$$Y = 0:\ w_0 = 1,\quad j^0 = -\frac{3}{2}\left[X + \frac{\partial w_0}{\partial Y}\right]\varepsilon^{1/2} + O(\varepsilon),$$

$$Y \to \infty:\ w_0 \to 0;\ Y > 0,\ X \to \infty:\ w_0 \to 0.$$

The boundary conditions at infinity represent the conditions for joining with the solution w = 0 in the outer region and with solution of (2.3) in the boundary layer of form $w_0 = A_0 \exp(-|X|Y)$ in the X and Y variables. Substitution of the variables $\eta = XY$, $\xi = [X^2 - (1 - e_r{}^0)Y^2]/2(1 - e_r{}^{02})^{1/2}$ changes (2.4) into the Klein–Gordon equation $\Delta w_0 = w_0$ on the (ξ, η) plane, whose solution with consideration of the boundary conditions is represented in the form

$$w_0(\xi, \eta) = \frac{1}{2\pi}\int_0^\infty \Psi(\tau) K_0([(\xi - \tau)^2 + \eta^2]^{1/2})\,d\tau; \tag{2.5}$$

$$\tau_0 \geqslant 0:\ \int_0^\infty \Psi(\tau) K_0(|\tau - \tau_0|)\,d\tau = 2\pi \tag{2.6}$$

[$K_0(x)$ is the MacDonald function].

The integral Fredholm equation of the first kind (2.6) is effectively solved by the Wiener–Hopf method [7]. Dropping the calculations, we will present the final solution

$$\Psi(\tau) = 2[\exp(-\tau)/(\pi\tau)^{1/2} + \operatorname{erf}\tau^{1/2}], \tag{2.7}$$

$$j^0 = \frac{3}{4}|X|\left[\Psi\left(\frac{1}{2}(1 - e^{02}_r)^{-1/2}X^2\right) - 2\operatorname{sgn}X\right]\varepsilon^{1/2} + O(\varepsilon).$$

Expressions (2.5) and (2.7) determine the function w and the local ionization current in the neighborhood of the critical point. The distribution of the volumetric charge is found by transition to the variable $q^* = 1 - w\exp(-Pe_E\varphi^*/2)$.

On satisfaction of the condition $||e_r{}^0| - 1| \leq O(\varepsilon^{2/3})$ in the neighborhood of the critical point of the electric field strength $(0 < r^* - 1 \leq O(\varepsilon^{1/3}),\ \sin\theta \leq O(\varepsilon^{1/3}))$ we have a different asymptote of the solution. The boundary-value problem obtained in this case for the zeroth approximation of the function w is written in the form

$$\frac{\partial^2 w_0}{\partial Y_1^2} + \frac{1}{X_1}\frac{\partial}{\partial X_1}\left(X_1\frac{\partial w_0}{\partial X_1}\right) = \left[\left(e^0_r + \frac{X_1^2}{2} - Y_1^2\right)^2 + X_1^2 Y_1^2\right] w_0, \tag{2.8}$$

$$X_1 \geqslant 0,\quad Y_1 \geqslant 0,$$

$$Y_1 \to \infty:\ w_0 \to 0;\ Y_1 > 0,\ X_1 \to \infty:\ w_0 \to 0,$$

$$Y_1 = 0:\ w_0 = 1,\quad j^0 = -\frac{3}{2}\left[\left(\frac{X_1^2}{2} + e^1_r\right)\operatorname{sgn} e^1_r - \frac{1}{2}\frac{\partial w}{\partial Y_1}\right]\varepsilon^{2/3} + O(\varepsilon),$$

$$X_1 = |\theta - \theta_0|\varepsilon^{-1/3},\ Y_1 = (r^* - 1)\varepsilon^{-1/3},\quad e_r^1 = (|e_r^0| - 1)\varepsilon^{-2/3},\qquad (2.8)$$
$$\theta_0 = \arccos(-\operatorname{sgn} e_r^0).$$

Equation (2.8) is numerically integrated to determine the electrification current and the volumetric charge near the critical point. As an example Fig. 1 shows the lines of levels 1-3, corresponding to values of $q^* = 0.1,\ 0.5,\ 0.9$ when $e_r^0 = -1$, $\theta_0 = \pi$.

On the basis of the calculations performed here and from (2.3) and (2.7) we have obtained the approximate analytical expressions for the particle charge current:

$$I^0 = F_\infty/3\mathrm{Pe}_E + \delta I^0,\qquad (2.9)$$
$$F_\infty = 3\mathrm{Pe}_E(I_1 + \varepsilon I_2 + \varepsilon^{4/3}I_3),$$
$$|e_r^0| < 1:\ I_1 = \frac{1}{4}(1 - e_r^0)^2,\quad I_2 = \frac{1}{4}(1 - e_r^{02})^{1/2},$$
$$|e_r^0| \geqslant 1:\ I_1 = \frac{1}{2}(\operatorname{sgn} e_r^0 - 1)e_r^0\quad I_2 = 0,$$
$$I_3 = [e_r^1/8(\exp 4{,}16e_r^1 - 1)]^{1/2} - f(e_r^1),$$
$$e_r^1 \geqslant 0:\ f = 0,\ e_r^1 < 0:\ f = (-e_r^1/8)^{1/2}.$$

The remaining term δI^0 has the order $O(\varepsilon^3)$ when $|e_r^0| \geq 1 + O(\varepsilon^{2/3})$, $O(\varepsilon^{3/2})$ for the case in which $|e_r^0| \leq 1 - O(\varepsilon^{2/3})$ and $O(\varepsilon^{5/3})$ when $||e_r^1| - 1| \leq O(\varepsilon^{2/3})$. The expression for I_3 represents an approximation of the function found as a result of the numerical solution of (2.8).

3. Let us limit ourselves to the further case of $K^* = \infty$ (the absolute absorbing surface of the particle). Using the method of joining the asymptotic expansions [5], Klett [4] found a solution for problem (1.1) for the case in which Pe_E was small and he derived the following expression for the global electrification current:

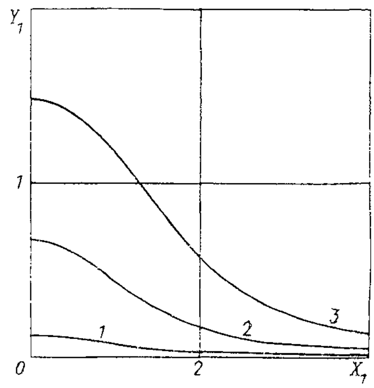

Fig. 1

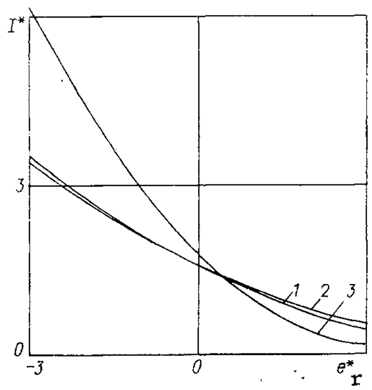

Fig. 2

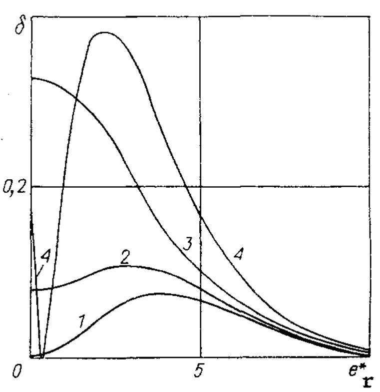

Fig. 3

$$I^* = F_0(\mathrm{Pe}_E, e_r^*) + O(\mathrm{Pe}_E), \tag{3.1}$$
$$F_0 = \Lambda(1 + \Lambda \exp e_r^* \mathrm{Pe}_E/2, \quad \Lambda(e_r^*) = e_r^*/(\exp e_r^* - 1),$$
$$e_r^* = 3\,\mathrm{Pe}_E\, e_r^0, \quad I^* = 3\,\mathrm{Pe}_E\, I^0.$$

Using the limit relationship (2.9) and (3.1), we can construct the approximate interpolation formula for the calculation of the charge current $I^* = I_a^*(\mathrm{Pe}_E, e_r^*)$ for the case in which $\mathrm{Pe}_E \sim 1$:

$$\mathrm{Pe}_E \leqslant 2\colon\ I_a^* = F_0(\mathrm{Pe}_E \varkappa_1, e_r^*), \tag{3.2}$$
$$\mathrm{Pe}_E > 2\colon\ I_a^* = F_\infty(\mathrm{Pe}_E \varkappa_2, e_r^*),$$
$$\varkappa_1 = 1 + 0.157\,\mathrm{Pe}_E^{0.745}, \quad \varkappa_2 = 1 + 0.445\,\mathrm{Pe}_E^{-1.43}.$$

The structure of expression (3.2) ensures satisfaction of the asymptotes of (2.9) and (3.1) in the limit cases $\mathrm{Pe}_E \to \infty$ and $\mathrm{Pe}_E \to 0$. The functions $\kappa_{1,2}(\mathrm{Pe}_E)$ have been selected so as to approximate the function $I_a(\mathrm{Pe}_E, e_r^*)$ to the function $I^*(\mathrm{Pe}_E, e_r^*)$, derived from the numerical solution of boundary-value problem (1.1) [8]. Good approximation is achieved here. The relative error $\delta_r(\mathrm{Pe}_E, e_r^*) = |I^* - I_a^*|/I^*$ for negative and moderately positive values of e_r^* does not exceed 10%. With an increase in e_r^*, $\delta_r(\mathrm{Pe}_E, e_r^*)$ increases because of the rapid diminution of I^*; however, the absolute error $|I^* - I_a^*| = \delta$ diminishes. As an example, Fig. 2 shows graphs of the relationship between the dimensionless electrification current and the particle charge, constructed for the case in which $\mathrm{Pe}_E = 1$ on the basis of formula (3.2) and from the results of the numerical calculation (curves 1 and 2).

Because of an absence of reliable interpolation formulas to calculate the electrification current in the case of a combined particle charge, until very recently we have used the simple relationship $I_s^*(\mathrm{Pe}_E, e_r^*) = 3\,\mathrm{Pe}_E\, I_1(\mathrm{Pe}_E, e_r^*) + \Lambda(e_r^*)$ (line 3 in Fig. 2), found through the simple addition of the principal terms of asymptotic expansions (2.9) and (3.1) in the limit cases $\mathrm{Pe}_E \to \infty$ and $\mathrm{Pe}_E \to 0$. However, as is easily seen, the principal terms of the expansions of the functions I_s^* and I^* (2.9) when $e_r^* < 0$, $\mathrm{Pe}_E \to \infty$ do not coincide. However, with moderate Pe_E values the charge current calculated in this fashion may differ by a factor of 2 or more from its true value. Moreover, analysis of Eqs. (1.1) and the function $I_s^*(\mathrm{Pe}_E, e_r^*)$ shows that for any values of Pe_E the relationship $e_r^* \to -\infty$: $I^* = -e_r^* + o(1)$, $I_s^* = -2e_r^* + o(1)$ is satisfied. In this connection, the greatest errors in the calculation of the particle electrification are to be expected in the case of bipolar charging of the dispersion phase, when the charge of each kind of ion is described by means of the function $I_s^*(\mathrm{Pe}_E, e_r^*)$ [9]. Formula (3.2) shows none of these shortcomings.

Figure 3 shows the absolute error arising through the utilization of formula (3.2), as well as the asymptotic expressions (3.1) and (2.9) (lines 1-3) as functions of the particle charge when $e_r^* > 0$ and $\mathrm{Pe}_E = 1$. Comparison shows that (2.9) and (3.1) for the approximate calculation can be used even with moderate Peclet numbers such as, for example, when $\mathrm{Pe}_E > 2$ and $\mathrm{Pe}_E \leq 2$, respectively. The magnitude of the error in the calculation of the electrification current in this case is somewhat greater than with utilization of (3.2), but is nevertheless substantially smaller than when we determine the current from the relationship $I_s^*(\mathrm{Pe}_E, e_r^*)$ (line 4 in Fig. 3).

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ACOUSTIC CERENKOV RADIATION AND ITS UTILIZATION IN HOLOGRAPHY METHODS TO STUDY MOVING MEDIA

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UDC 534.222:532.574

An interference interpretation is presented for acoustic Cerenkov radiation and we consider the possibility of its utilization as a basis of holography methods to study moving media.

In [1, 2] we find an examination of the possibility of using the sound scattering effect to find the average velocity and distribution of velocities for combustion products on the basis of the cross-sectional area of a combustion chamber. A new phenomenon has been studied rather recently, and namely, the acoustic Cerenkov radiation [3, 4] and some of its aspects of application, in particular, the utilization of this effect in solving problems analogous to those presented in [2].

In the present paper we put forward an "interference" interpretation of acoustic Cerenkov radiation and we examine a new approach to the solution of problems (analogous to those mentioned above), based on the utilization of the Cerenkov radiation and the methods of dynamic holography [5-9]. These questions are fundamental, both from the standpoint of validating the possibilities of using this new approach to the solution of numerous applied problems, as well as in connection with the fact that the solution that we will present later on is subsequently necessary for a more detailed and penetrating investigation into the process of holography, in the processing and deciphering of the results, etc.

The methods and means presently at hand to determine the parameters of moving media (in particular, products of combustion) frequently are of inadequate accuracy, and the resulting information, as a rule, is both insufficient and fails to provide a complete picture of the phenomena being studied. The latter, in turn, is one of the reasons why the determination of the values for the parameters of the medium are determined with unsatisfactory accuracy.

In our opinion, an extremely promising approach, in terms of the completeness of information and its accuracy, involves the methods of dynamic holography in combination with acoustic Cerenkov radiation. In particular, this approach may prove to be useful in studying nonsteady and fast-moving processes, since the information about the medium may be developed virtually instantaneously and it will exhibit four-dimensional characteristics (the three coordinates and time). Moreover, we have the possibility of contact-free (i.e., performed on the outside surface of the wall, or of the combustion chamber) probing in which the probe region is situated at a considerable distance from the point of "egress" for the combustion products from the combustion chamber, with the information being transmitted to the outside at a controlled Cerenkov angle.

The use of acoustic Cerenkov radiation expands the possibilities of dynamic holography, i.e., new channels for the transmission of information are created (with the Cerenkov angle), and the hologram itself will differ from the familiar features encountered in the supersonic (or in faster-than-light with electromagnetic radiation) motion of an interference grid. Moreover, the information on the medium (the object) is obtained virtually instantaneously,

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